2
Relations

ORDERED PAIR *Two numbers a and b listed in a specific order and enclosed in parentheses form an ordered pair* (a, b).

In the ordered pair (a, b), we call a as *first member* (or *first component*) and b as *second member* (or *second component*).

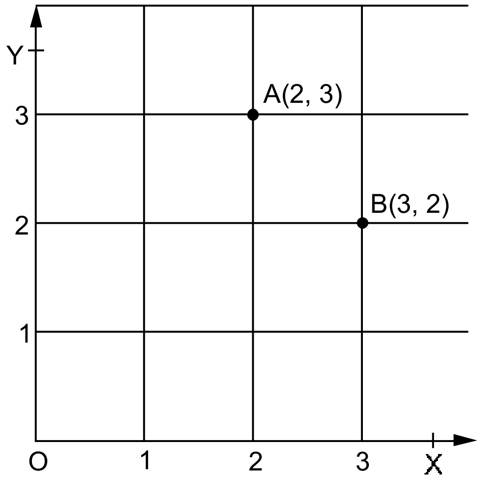
By interchanging the positions of the components, the ordered pair is changed.

Thus, $(a, b) \neq (b, a)$.

EXAMPLE In coordinate geometry, the position of a point in a plane is determined by an ordered pair.

In the given figure, the ordered pairs (2, 3) and (3, 2) represent two different points A and B respectively.

Thus, $(2, 3) \neq (3, 2)$.



EQUALITY OF TWO ORDERED PAIRS

We have $(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$.

EXAMPLE 1 *Find a and b, when* $(a - 1, b + 5) = (2, 3)$.

SOLUTION Using the definition of equality of two ordered pairs, we have

$$(a - 1, b + 5) = (2, 3) \Rightarrow a - 1 = 2 \text{ and } b + 5 = 3$$
$$\Rightarrow a = 3 \text{ and } b = -2.$$

Hence, $a = 3$ and $b = -2$.

EXAMPLE 2 *Find a and b, when* $(2a + b, 11) = (1, a - 3b)$.

SOLUTION Using the definition of equality of two ordered pairs, we have

$$(2a + b, 11) = (1, a - 3b)$$

$$\Rightarrow \begin{cases} 2a + b = 1 & \text{... (i)} \\ a - 3b = 11. & \text{... (ii)} \end{cases}$$

On solving (i) and (ii), we get $a = 2$ and $b = -3$.

Hence, $a = 2$ and $b = -3$.

EXAMPLE 3 *If* $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, *find the values of x and y.*

SOLUTION Since the given two ordered pairs are equal, we have

$$\frac{x}{3}+1=\frac{5}{3} \Rightarrow \frac{x}{3}=\left(\frac{5}{3}-1\right)=\frac{2}{3} \Rightarrow \frac{x}{3}=\frac{2}{3} \Rightarrow x=2$$

and $y-\frac{2}{3}=\frac{1}{3} \Rightarrow y=\left(\frac{1}{3}+\frac{2}{3}\right)=\frac{3}{3}=1 \Rightarrow y=1.$

Hence, $x = 2$ and $y = 1$.

EXAMPLE 4 *Express* $\{(x, y) : x^2 + y^2 = 25$, *where* $x, y \in W\}$ *as a set of ordered pairs.*

SOLUTION It is easy to verify that each of the following ordered pairs of whole numbers satisfies the given relation $x^2 + y^2 = 25$:

$(5, 0), (0, 5), (3, 4)$ and $(4, 3)$.

Hence, the set of required ordered pairs is

$\{(5, 0), (0, 5), (3, 4), (4, 3)\}$.

CARTESIAN PRODUCT OF TWO SETS

Let A and B be two nonempty sets. Then, the Cartesian product of A and B is the set denoted by $(A \times B)$, *consisting of all ordered pairs (a, b) such that* $a \in A$ *and* $b \in B$.

$\therefore \quad A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

If $A = \phi$ or $B = \phi$, we define $A \times B = \phi$.

REMARKS (i) *If* $n(A) = p$ *and* $n(B) = q$ *then* $n(A \times B) = pq$ *and* $n(B \times A) = pq$.

(ii) *If at least one of A and B is infinite then* $(A \times B)$ *is infinite and* $(B \times A)$ *is infinite.*

SOLVED EXAMPLES

EXAMPLE 1 *If* $A = \{1, 3, 5\}$ *and* $B = \{2, 3\}$ *then find:*

(i) $A \times B$ *(ii)* $B \times A$ *(iii)* $(A \times B) \cap (B \times A)$

SOLUTION We have

(i) $A \times B = \{1, 3, 5\} \times \{2, 3\}$

$= \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$.

(ii) $B \times A = \{2, 3\} \times \{1, 3, 5\}$

$= \{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$.

(iii) $(A \times B) \cap (B \times A) = \{(3, 3)\}$.

EXAMPLE 2 *If* $A = \{1, 2, 3\}, B = \{3, 4\}$ *and* $C = \{4, 5, 6\}$ *then find:*

(i) $A \times (B \cap C)$ *(ii)* $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$ *(iv)* $(A \times B) \cup (A \times C)$

SOLUTION We have

(i) $B \cap C = \{3, 4\} \cap \{4, 5, 6\} = \{4\}$.

$\therefore \quad A \times (B \cap C) = \{1, 2, 3\} \times \{4\} = \{(1, 4), (2, 4), (3, 4)\}$.

(ii) $(A \times B) = \{1, 2, 3\} \times \{3, 4\}$

$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$.

$(A \times C) = \{1, 2, 3\} \times \{4, 5, 6\}$

$= \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

$\therefore \quad (A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$.

(iii) $B \cup C = \{3, 4\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$.

$\therefore \quad A \times (B \cup C) = \{1, 2, 3\} \times \{3, 4, 5, 6\}$

$= \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$.

(iv) Also, from (ii), we get

$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$.

EXAMPLE 3 *Let* $A = \{x \in N : x^2 - 5x + 6 = 0\}$, $B = \{x \in W : 0 \le x < 2\}$ *and* $C = \{x \in N : x < 3\}$. *Verify that*

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

SOLUTION We have

$A = \{x \in N : x^2 - 5x + 6 = 0\} = \{x \in N : (x-2)(x-3) = 0\} = \{2, 3\}$;

$B = \{x \in W : 0 \le x < 2\} = \{0, 1\}$ and $C = \{x \in N : x < 3\} = \{1, 2\}$.

$\therefore \quad A = \{2, 3\}, B = \{0, 1\}$ and $C = \{1, 2\}$.

(i) $(B \cup C) = \{0, 1\} \cup (1, 2\} = \{0, 1, 2\}$.

$\therefore \quad A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$.

Now, $(A \times B) = \{2, 3\} \times \{0, 1\}$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

and $\quad (A \times C) = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$.

$\therefore \quad (A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$.

Hence, $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(ii) $(B \cap C) = \{0, 1\} \cap \{1, 2\} = \{1\}$.

$\therefore \quad A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$.

And,

$(A \times B) \cap (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 1), (3, 1)\}$.

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

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EXAMPLE 4 *If* $(A \times B) = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$, *find A and B.*

SOLUTION Clearly, we have

A = set of all first coordinates of the elements of $(A \times B)$

$= \{3, 5\}$.

B = set of all second coordinates of the elements of $(A \times B)$

$= \{2, 4\}$.

Thus, $A = \{3, 5\}$ and $B = \{2, 4\}$.

EXAMPLE 5 *A and B are two sets given in such a way that* $(A \times B)$ *contains 6 elements. If three elments of* $(A \times B)$ *be* (1, 3), (2, 5) *and* (3, 3), *find its remaining elements.*

SOLUTION Since (1, 3), (2, 5) and (3, 3) are in $(A \times B)$, it follows that 1, 2, 3 are elements of *A* and 3, 5 are elements of *B*.

$\therefore \quad (A \times B) = \{1, 2, 3\} \times \{3, 5\}$

$= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$.

Hence, the remaining elements of $(A \times B)$ are (1, 5), (2, 3) and (3, 5).

EXAMPLE 6 *If* $A = \{a, b\}$, *find* $(A \times A)$.

SOLUTION We have

$(A \times A) = \{a, b\} \times \{a, b\}$

$= \{(a, a), (a, b), (b, a), (b, b)\}$.

EXAMPLE 7 *Let R be the set of all real numbers. What does* $(R \times R)$ *represent?*

SOLUTION We have $(R \times R) = \{(x, y) : x, y \in R\}$.

Thus, $(R \times R)$ represents the set of all coordinates of points in two-dimensional space.

EXAMPLE 8 *If* $(A \times A)$ *has 9 elements two of which are* $(-1, 0)$ *and* $(0, 1)$, *find the set A and the remaining elements of* $(A \times A)$.

SOLUTION Clearly, -1, 0 and 1 are elements of *A*.

$\therefore \quad A = \{-1, 0, 1\}$

$A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$

$= \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$.

Hence, the remaining elements of $(A \times A)$ are

$(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$.

GRAPHICAL REPRESENTATION OF $A \times B$ AND $B \times A$

Let $X'OX$ and YOY' be the *x*-axis and *y*-axis respectively, drawn on a graph paper.

Let $A = \{-1, 2, 4\}$ and $B = \{1, 3\}$. Then,

$A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (4, 1), (4, 3)\}$.

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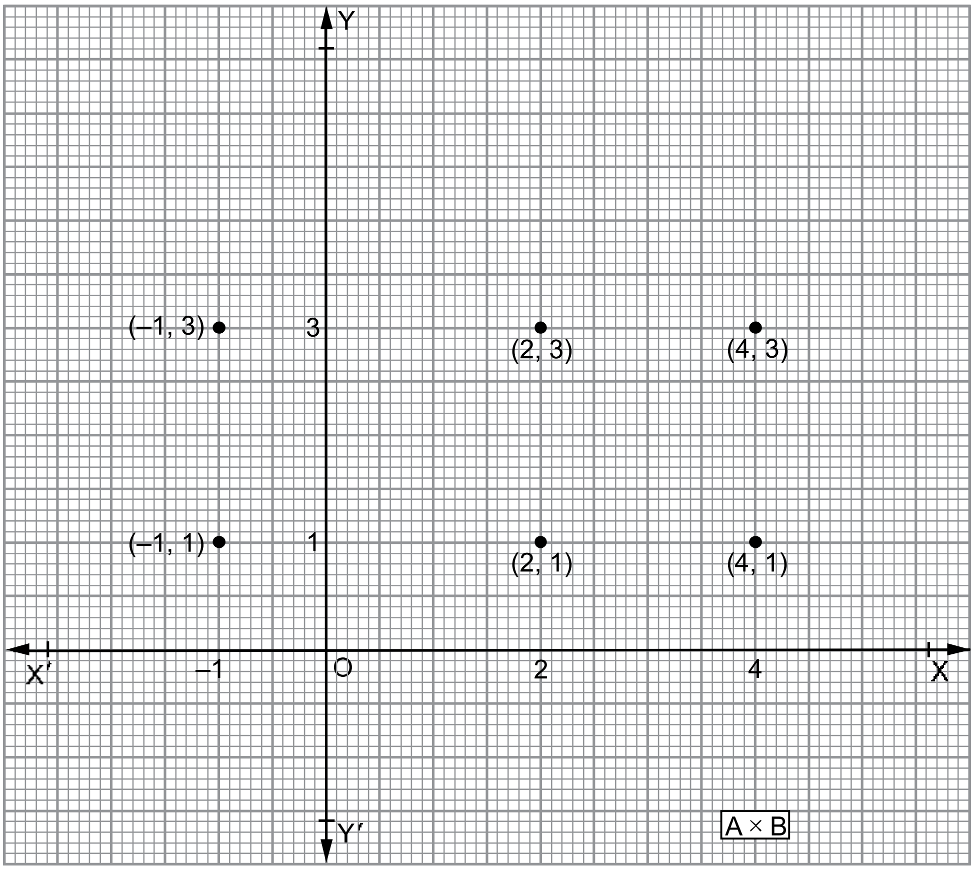
These points may be plotted on the graph paper, as shown below:



This is the graphical representation of $A \times B$.

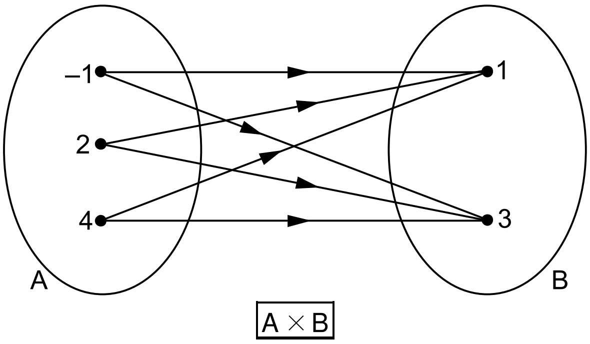
For $B \times A$, we take $B = \{1, 3\}$ along the *x*-axis and $A = \{-1, 2, 4\}$ along the *y*-axis and plot the points $(1, -1), (1, 2), (1, 4), (3, -1), (3, 2), (3, 4)$ on the graph paper.

Arrow diagram of $\boldsymbol{A \times B}$

Let $A = \{-1, 2, 4\}$ and $B = \{1, 3\}$. Then,

$$A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (4, 1), (4, 3)\}.$$

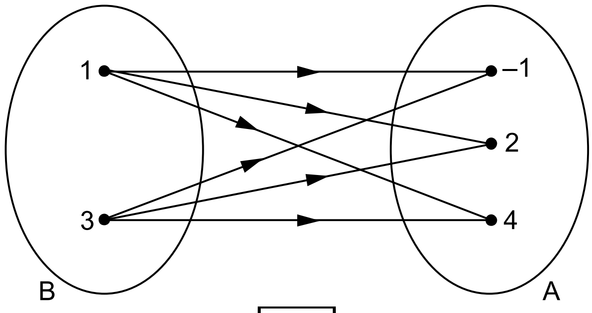
Then, $A \times B$ may be represented by arrow diagram, as shown below:



Arrow diagram of $\boldsymbol{B \times A}$

Again, $B \times A = \{(1, -1), (1, 2), (1, 4), (3, -1), (3, 2), (3, 4)\}$.

We may exhibit $B \times A$ by arrow diagram, as shown below:



B × A

ORDERED TRIPLET Three numbers *a, b* and *c* listed in a specific order and enclosed in parentheses form an ordered triplet (a, b, c).

Thus, $(1, 2, 3) \neq (2, 1, 3) \neq (3, 2, 1)$, etc.

For any nonempty set *A*, we define:

$$(A \times A \times A) = \{(a, b, c) : a, b, c \in A\}.$$

EXAMPLE 9 *If* $A = \{1, 2\}$, *find* $(A \times A \times A)$.

SOLUTION $A \times A = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$.

$$\begin{aligned}\therefore \quad A \times A \times A &= (A \times A) \times A \\ &= \{(1, 1), (1, 2), (2, 1), (2, 2)\} \times \{1, 2\} \\ &= \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), \\ &\qquad (2, 1, 2), (2, 2, 1), (2, 2, 2)\}.\end{aligned}$$

EXAMPLE 10 *Let R be the set of all real numbers. What does* $(R \times R \times R)$ *represent?*

SOLUTION $(R \times R \times R) = \{(a, b, c) : a, b, c \in R\}$.

Thus, $(R \times R \times R)$ represents the set of all points in three-dimensional space.

REMARK For any nonempty sets *A*, *B* and *C*, we always have

$$(A \times B) \times C = A \times (B \times C) = A \times B \times C.$$

EXAMPLE 11 *Let* $A = \{1, 2\}$, $B = \{3, 4\}$ *and* $C = \{4, 5\}$.

Verify that $(A \times B) \times C = A \times (B \times C)$ *and hence find* $A \times B \times C$.

SOLUTION We have

$$A \times B = \{1, 2\} \times \{3, 4\} = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$\begin{aligned}\Rightarrow \quad (A \times B) \times C &= \{(1, 3), (1, 4), (2, 3), (2, 4)\} \times \{4, 5\} \\ &= \{(1, 3, 4), (1, 3, 5), (1, 4, 4), (1, 4, 5), (2, 3, 4), \\ &\qquad (2, 3, 5), (2, 4, 4), (2, 4, 5)\}.\end{aligned}$$

Again, $B \times C = \{3, 4\} \times \{4, 5\} = \{(3, 4), (3, 5), (4, 4), (4, 5)\}$

$$\begin{aligned}\Rightarrow \quad A \times (B \times C) &= \{1, 2\} \times \{(3, 4), (3, 5), (4, 4), (4, 5)\} \\ &= \{(1, 3, 4), (1, 3, 5), (1, 4, 4), (1, 4, 5), (2, 3, 4), \\ &\qquad (2, 3, 5), (2, 4, 4), (2, 4, 5)\}.\end{aligned}$$

$$\therefore \quad (A \times B) \times C = A \times (B \times C) = A \times B \times C.$$

Hence, $(A \times B \times C) = \{(1, 3, 4), (1, 3, 5), (1, 4, 4), (1, 4, 5), (2, 3, 4), (2, 3, 5), (2, 4, 4), (2, 4, 5)\}$.

EXERCISE 2A

1. Find the values of *a* and *b*, when:
   (i) $(a+3, b-2) = (5, 1)$ (ii) $(a+b, 2b-3) = (4, -5)$
   (iii) $\left(\frac{a}{3}+1, b-\frac{1}{3}\right) = \left(\frac{5}{3}, \frac{2}{3}\right)$ (iv) $(a-2, 2b+1) = (b-1, a+2)$
2. If $A = \{0, 1\}$ and $B = \{1, 2, 3\}$, show that $A \times B \neq B \times A$.
3. If $P = \{a, b\}$ and $Q = \{x, y, z\}$, show that $P \times Q \neq Q \times P$.
4. If $A = \{2, 3, 5\}$ and $B = \{5, 7\}$, find:
   (i) $A \times B$ (ii) $B \times A$ (iii) $A \times A$ (iv) $B \times B$
5. If $A = \{x \in N : x \leq 3\}$ and $B = \{x \in W, x < 2\}$, find $(A \times B)$ and $(B \times A)$. Is $(A \times B) = (B \times A)$?
6. If $A = \{1, 3, 5\}$, $B = \{3, 4\}$ and $C = \{2, 3\}$, verify that:
   (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
7. Let $A = \{x \in W : x < 2\}$, $B = \{x \in N : 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that:
   (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

   Hint $A = \{0, 1\}$, $B = \{2, 3, 4\}$ and $C = \{3, 5\}$.
8. If $A \times B = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$, find *A* and *B*.
9. Let $A = \{2, 3\}$ and $B = \{4, 5\}$. Find $(A \times B)$. How many subsets will $(A \times B)$ have?
10. Let $A \times B = \{(a, b) : b = 3a - 2\}$. If $(x, -5)$ and $(2, y)$ belong to $A \times B$, find the values of *x* and *y*.
11. Let *A* and *B* be two sets such that $n(A) = 3$ and $n(B) = 2$.
    If $a \neq b \neq c$ and $(a, 0), (b, 1), (c, 0)$ are in $A \times B$, find *A* and *B*.
12. Let $A = \{-2, 2\}$ and $B = \{0, 3, 5\}$. Find:
    (i) $A \times B$ (ii) $B \times A$ (iii) $A \times A$ (iv) $B \times B$
    Represent each of the above (a) graphically and (b) by arrow diagram.
13. If $A = \{5, 7\}$, find (i) $A \times A$ and (ii) $A \times A \times A$.
14. Let $A = \{-3, -1\}$, $B = \{1, 3\}$ and $C = \{3, 5\}$. Find:
    (i) $A \times B$ (ii) $(A \times B) \times C$ (iii) $B \times C$ (iv) $A \times (B \times C)$

ANSWERS (EXERCISE 2A)

1. (i) $a = 2, b = 3$ (ii) $a = 5, b = -1$ (iii) $a = 2, b = 1$ (iv) $a = 3, b = 2$
4. (i) $A \times B = \{(2, 5), (2, 7), (3, 5), (3, 7), (5, 5), (5, 7)\}$
   (ii) $B \times A = \{(5, 2), (5, 3), (5, 5), (7, 2), (7, 3), (7, 5)\}$
   (iii) $A \times A = \{(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)\}$
   (iv) $B \times B = \{(5, 5), (5, 7), (7, 5), (7, 7)\}$

5. $A \times B = \{(1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1)\}$;
$B \times A = \{(0, 1), (0, 2), (0, 3), (1, 1), (1, 2), (1, 3)\}$;
No

8. $A = \{-2, 0, 3\}$ and $B = \{3, 4\}$

9. $A \times B = \{(2, 4), (2, 5), (3, 4), (3, 5)\}$; Number of subsets of $(A \times B) = 2^4 = 16$

10. $x = -1, y = 4$

11. $A = \{a, b, c\}$ and $B = \{0, 1\}$

12. (i) $A \times B = \{(-2, 0), (-2, 3), (-2, 5), (2, 0), (2, 3), (2, 5)\}$
(ii) $B \times A = \{(0, -2), (0, 2), (3, -2), (3, 2), (5, -2), (5, 2)\}$
(iii) $A \times A = \{(-2, -2), (-2, 2), (2, -2), (2, 2)\}$
(iv) $B \times B = \{(0, 0), (0, 3), (0, 5), (3, 0), (3, 3), (3, 5), (5, 0), (5, 3), (5, 5)\}$

13. (i) $A \times A = \{(5, 5), (5, 7), (7, 5), (7, 7)\}$
(ii) $A \times A \times A = \{(5, 5, 5), (5, 7, 5), (7, 5, 5), (7, 7, 5), (5, 5, 7), (5, 7, 7), (7, 5, 7), (7, 7, 7)\}$

14. (i) $A \times B = \{(-3, 1), (-3, 3), (-1, 1), (-1, 3)\}$
(ii) $(A \times B) \times C = \{(-3, 1, 3), (-3, 3, 3), (-1, 1, 3), (-1, 3, 3), (-3, 1, 5), (-3, 3, 5), (-1, 1, 5), (-1, 3, 5)\}$
(iii) $B \times C = \{(1, 3), (1, 5), (3, 3), (3, 5)\}$
(iv) $A \times (B \times C) = \{(-3, 1, 3), (-3, 1, 5), (-3, 3, 3), (-3, 3, 5), (-1, 1, 3), (-1, 1, 5), (-1, 3, 3), (-1, 3, 5)\}$

SOME USEFUL RESULTS

THEOREM 1 *If $A \subseteq B$ then prove that $A \times C \subseteq B \times C$ for any set C.*

PROOF Let $A \subseteq B$ and let (*a, c*) be an arbitrary element of $A \times C$. Then,

$$(a, c) \in A \times C \Rightarrow a \in A \text{ and } c \in C$$
$$\Rightarrow a \in B \text{ and } c \in C \qquad [\because A \subseteq B]$$
$$\Rightarrow (a, c) \in B \times C.$$

Thus, every element of $A \times C$ is contained in $B \times C$.

$\therefore \quad A \times C \subseteq B \times C.$

Hence, $A \subseteq B \Rightarrow A \times C \subseteq B \times C$ for any set *C*.

THEOREM 2 *If $A \subseteq B$ and $C \subseteq D$ then prove that $A \times C \subseteq B \times D$.*

PROOF Let $A \subseteq B$ and $C \subseteq D$. Then, we have to show that $A \times C \subseteq B \times D$.

Let (*a, c*) be an arbitrary element of $A \times C$. Then,

$$(a, c) \in A \times C \Rightarrow a \in A \text{ and } c \in C$$
$$\Rightarrow a \in B \text{ and } c \in D \qquad [\because A \subseteq B \text{ and } C \subseteq D]$$
$$\Rightarrow (a, c) \in B \times D.$$

Thus, every element of $A \times C$ is contained in $B \times D$.

Hence, $A \times C \subseteq B \times D$.

THEOREM 3 *If A and B are any two nonempty sets, prove that* $A \times B = B \times A \Leftrightarrow A = B$.

PROOF Let us first assume that $A = B$. Then,

$$A \times B = B \times B \text{ and } B \times A = B \times B \qquad [\because A = B].$$

$\therefore \quad A \times B = B \times A$.

Thus, $A = B \Rightarrow A \times B = B \times A$.

Conversely, let $A \times B = B \times A$ and we have to show that $A = B$.

Let *a* be an arbitrary element of *A*. Then,

$$\begin{aligned} a \in A &\Rightarrow (a, b) \in A \times B \text{ for some } b \in B \\ &\Rightarrow (a, b) \in B \times A \qquad [\because A \times B = B \times A] \\ &\Rightarrow a \in B \text{ and } b \in A \\ &\Rightarrow a \in B \quad \text{(surely).} \end{aligned}$$

$\therefore \quad A \subseteq B$.

Again, let *b* be an arbitrary element of *B*. Then,

$$\begin{aligned} b \in B &\Rightarrow (b, a) \in B \times A \text{ for some } a \in A \\ &\Rightarrow (b, a) \in A \times B \qquad [\because B \times A = A \times B] \\ &\Rightarrow b \in A \text{ and } a \in B \\ &\Rightarrow b \in A \quad \text{(surely).} \end{aligned}$$

$\therefore \quad B \subseteq A$.

Thus, $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$.

$\therefore \quad A \times B = B \times A \Rightarrow A = B$.

Hence, $A \times B = B \times A \Leftrightarrow A = B$.

THEOREM 4 *If* $A \subseteq B$, *prove that* $A \times A \subseteq (A \times B) \cap (B \times A)$.

PROOF Let $A \subseteq B$ and let (*a*, *a*) be an arbitrary element of $A \times A$.

Then,
$$\begin{aligned} (a, a) \in A \times A &\Rightarrow a \in A \text{ and } a \in A \\ &\Rightarrow a \in A \text{ and } a \in B \qquad [\because A \subseteq B] \\ &\Rightarrow (a \in A, a \in B) \text{ and } (a \in B, a \in A) \\ &\Rightarrow (a, b) \in A \times B \text{ and } (a, b) \in B \times A \\ &\Rightarrow (a, b) \in (A \times B) \cap (B \times A). \end{aligned}$$

$\therefore \quad A \times A \subseteq (A \times B) \cap (B \times A)$.

Hence, $A \subseteq B \Rightarrow A \times A \subseteq (A \times B) \cap (B \times A)$.

THEOREM 5 *For any sets A, B and C, prove that*

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

PROOF Let (*a*, *b*) be an arbitrary element of $A \times (B \cup C)$. Then,

$$\begin{aligned} (a, b) \in A \times (B \cup C) &\Rightarrow a \in A \text{ and } b \in B \cup C \\ &\Rightarrow a \in A \text{ and } (b \in B \text{ or } b \in C) \\ &\Rightarrow (a \in A \text{ and } b \in B) \text{ or } (a \in A \text{ and } b \in C) \\ &\qquad [\because p \text{ and } (q \text{ or } r) \equiv (p \text{ and } q) \text{ or } (p \text{ and } r)] \\ &\Rightarrow (a, b) \in A \times B \text{ or } (a, b) \in A \times C \\ &\Rightarrow (a, b) \in (A \times B) \cup (A \times C). \end{aligned}$$

$\therefore \quad A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$. ... (i)

Again, let (x, y) be an arbitrary element of $(A \times B) \cup (A \times C)$. Then,

$$\begin{aligned}(x, y) \in (A \times B) \cup (A \times C) &\Rightarrow (x, y) \in A \times B \text{ or } (x, y) \in A \times C \\ &\Rightarrow (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C) \\ &\Rightarrow x \in A \text{ and } (y \in B \text{ or } y \in C) \\ &\qquad \text{[by distributive law]} \\ &\Rightarrow x \in A \text{ and } y \in (B \cup C) \\ &\Rightarrow (x, y) \in A \times (B \cup C).\end{aligned}$$

$\therefore \quad (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$. ... (ii)

Hence, from (i) and (ii), we get

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

THEOREM 6 *For any sets A, B and C, prove that*

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

PROOF Let (a, b) be an arbitrary element of $A \times (B \cap C)$. Then,

$$\begin{aligned}(a, b) \in A \times (B \cap C) &\Rightarrow a \in A \text{ and } b \in (B \cap C) \\ &\Rightarrow a \in A \text{ and } (b \in B \text{ and } b \in C) \\ &\Rightarrow (a \in A \text{ and } b \in B) \text{ and } (a \in A \text{ and } b \in C) \\ &\Rightarrow (a, b) \in A \times B \text{ and } (a, b) \in A \times C \\ &\Rightarrow (a, b) \in (A \times B) \cap (A \times C).\end{aligned}$$

$\therefore \quad A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$. ... (i)

Again, let (x, y) be an arbitrary element of $(A \times B) \cap (A \times C)$. Then,

$$\begin{aligned}(x, y) \in (A \times B) \cap (A \times C) &\Rightarrow (x, y) \in A \times B \text{ and } (x, y) \in A \times C \\ &\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C) \\ &\Rightarrow x \in A \text{ and } (y \in B \text{ and } y \in C) \\ &\Rightarrow x \in A \text{ and } y \in (B \cap C) \\ &\Rightarrow (x, y) \in A \times (B \cap C).\end{aligned}$$

$\therefore \quad (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$. ... (ii)

Hence, from (i) and (ii), we get

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

THEOREM 7 *For any sets A, B and C, prove that*

$$A \times (B - C) = (A \times B) - (A \times C).$$

PROOF Let (a, b) be an arbitrary element of $A \times (B - C)$. Then,

$$\begin{aligned}(a, b) \in A \times (B - C) &\Rightarrow a \in A \text{ and } b \in (B - C) \\ &\Rightarrow a \in A \text{ and } (b \in B \text{ and } b \notin C) \\ &\Rightarrow (a \in A \text{ and } b \in B) \text{ and } (a \in A \text{ and } b \notin C) \\ &\Rightarrow (a, b) \in A \times B \text{ and } (a, b) \notin A \times C \\ &\Rightarrow (a, b) \in (A \times B) - (A \times C).\end{aligned}$$

$\therefore \quad A \times (B - C) \subseteq (A \times B) - (A \times C)$. ... (i)

Again, let (x, y) be an arbitrary element of $(A \times B) - (A \times C)$. Then,

$$\begin{aligned}(x, y) \in (A \times B) - (A \times C) &\Rightarrow (x, y) \in A \times B \text{ and } (x, y) \notin A \times C\\ &\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \notin C)\\ &\Rightarrow x \in A \text{ and } (y \in B \text{ and } y \notin C)\\ &\Rightarrow x \in A \text{ and } y \in (B - C)\\ &\Rightarrow (x, y) \in A \times (B - C).\end{aligned}$$

$\therefore \quad (A \times B) - (A \times C) \subseteq A \times (B - C)$. ... (ii)

From (i) and (ii), we get

$A \times (B - C) = (A \times B) - (A \times C)$.

THEOREM 8 *For any sets A, B, C and D, prove that*

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$

PROOF Let (a, b) be an arbitrary element of $(A \times B) \cap (C \times D)$. Then,

$$\begin{aligned}(a, b) \in (A \times B) \cap (C \times D) &\Rightarrow (a, b) \in (A \times B) \text{ and } (a, b) \in (C \times D)\\ &\Rightarrow (a \in A \text{ and } b \in B) \text{ and } (a \in C \text{ and } b \in D)\\ &\Rightarrow (a \in A \text{ and } a \in C) \text{ and } (b \in B \text{ and } b \in D)\\ &\Rightarrow a \in (A \cap C) \text{ and } b \in (B \cap D)\\ &\Rightarrow (a, b) \in (A \cap C) \times (B \cap D). \quad \text{... (i)}\end{aligned}$$

Again, let (x, y) be an arbitrary element of $(A \cap C) \times (B \cap D)$. Then,

$$\begin{aligned}(x, y) \in (A \cap C) \times (B \cap D) &\Rightarrow x \in A \cap C \text{ and } y \in B \cap D\\ &\Rightarrow (x \in A \text{ and } x \in C) \text{ and } (y \in B \text{ and } y \in D)\\ &\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in C \text{ and } y \in D)\\ &\Rightarrow (x, y) \in A \times B \text{ and } (x, y) \in C \times D\\ &\Rightarrow (x, y) \in (A \times B) \cap (C \times D).\end{aligned}$$

$\therefore \quad (A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$. ... (ii)

From (i) and (ii), we get

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

THEOREM 9 *For any sets A and B, prove that*

$$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A).$$

SOLUTION Let (a, b) be an arbitrary element of $(A \times B) \cap (B \times A)$. Then,

$$\begin{aligned}(a, b) \in (A \times B) \cap (B \times A) &\Rightarrow (a, b) \in A \times B \text{ and } (a, b) \in B \times A\\ &\Rightarrow (a \in A \text{ and } b \in B) \text{ and } (a \in B \text{ and } b \in A)\\ &\Rightarrow (a \in A \text{ and } a \in B) \text{ and } (b \in B \text{ and } b \in A)\\ &\Rightarrow a \in (A \cap B) \text{ and } b \in (B \cap A)\\ &\Rightarrow (a, b) \in (A \cap B) \times (B \cap A).\end{aligned}$$

$\therefore \quad (A \times B) \cap (B \times A) \subseteq (A \cap B) \times (B \cap A)$. ... (i)

Again, let (x, y) be an arbitrary element of $(A \cap B) \times (B \cap A)$. Then,

$$\begin{aligned}(x, y) \in (A \cap B) \times (B \cap A) &\Rightarrow x \in A \cap B \text{ and } y \in B \cap A\\ &\Rightarrow (x \in A \text{ and } x \in B) \text{ and } (y \in B \text{ and } y \in A)\end{aligned}$$

$\Rightarrow$ $(x \in A$ and $y \in B)$ and $(x \in B$ and $y \in A)$

$\Rightarrow$ $(x, y) \in A \times B$ and $(x, y) \in B \times A$

$\Rightarrow$ $(x, y) \in (A \times B) \cap (B \times A)$.

$\therefore$ $(A \cap B) \times (B \cap A) \subseteq (A \times B) \cap (B \times A)$. ... (ii)

Thus, from (i) and (ii), we get

$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$.

AN IMPORTANT RESULT *If A and B are two nonempty sets having n elements in common then $(A \times B)$ and $(B \times A)$ have n^2 elements in common.*

EXAMPLE *Let A and B be two nonempty sets such that $n(A) = 5$, $n(B) = 6$ and $n(A \cap B) = 3$.*

Find (i) $n(A \times B)$, (ii) $n(B \times A)$ and (iii) $n\{(A \times B) \cap (B \times A)\}$.

SOLUTION (i) $n(A \times B) = n(A) \times n(B) = (5 \times 6) = 30$.

(ii) $n(B \times A) = n(B) \times n(A) = (6 \times 5) = 30$.

(iii) Given: $n(A \cap B) = 3$.

$\therefore$ *A* and *B* have 3 elements in common.

So, $(A \times B)$ and $(B \times A)$ have $3^2 = 9$ elements in common.

Hence, $n\{(A \times B) \cap (B \times A)\} = 9$.

THEOREM 10 *If A, B and C be three nonempty sets given in such a way that $A \times B = A \times C$ then prove that $B = C$.*

PROOF Let $A \times B = A \times C$ and we have to prove that $B = C$.

Let $b \in B$. Then,

$b \in B \Rightarrow (a, b) \in A \times B$ for every $a \in A$

$\Rightarrow (a, b) \in A \times C$ for every $a \in A$ $[\because A \times B = A \times C]$

$\Rightarrow b \in C$.

$\therefore$ $B \subseteq C$. ... (i)

Again, let $c \in C$. Then,

$c \in C \Rightarrow (a, c) \in A \times C$ for every $a \in A$

$\Rightarrow (a, c) \in A \times B$ for every $a \in A$ $[\because A \times C = A \times B]$

$\Rightarrow c \in B$.

$\therefore$ $C \subseteq B$. ... (ii)

From (i) and (ii), we get $B = C$.

Hence, $A \times B = A \times C \Rightarrow B = C$.

EXERCISE 2B

1. For any sets *A*, *B* and *C*, prove that:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

2. For any sets A and B, prove that
$$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A).$$
3. If A and B are nonempty sets, prove that
$$A \times B = B \times A \Leftrightarrow A = B.$$
4. (i) If $A \subseteq B$, prove that $A \times C \subseteq B \times C$ for any set C.
   (ii) If $A \subseteq B$ and $C \subseteq D$ then prove that $A \times C \subseteq B \times D$.
5. If $A \times B \subseteq C \times D$ and $A \times B \neq \phi$, prove that $A \subseteq C$ and $B \subseteq D$.
6. If A and B be two sets such that $n(A) = 3$, $n(B) = 4$ and $n(A \cap B) = 2$ then find:
   (i) $n(A \times B)$ (ii) $n(B \times A)$ (iii) $n\{(A \times B) \cap (B \times A\}$
7. For any two sets A and B, show that $A \times B$ and $B \times A$ have an element in common if and only if A and B have an element in common.
8. Let $A = \{1, 2\}$ and $B = \{2, 3\}$. Then, write down all possible subsets of $A \times B$.
9. Let $A = \{a, b, c, d\}$, $B = \{c, d, e\}$ and $C = \{d, e, f, g\}$. Then verify each of the following identities:
   (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (ii) $A \times (B - C) = (A \times B) - (A \times C)$
   (iii) $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$

ANSWERS (EXERCISE 2B)

6. (i) $n(A \times B) = 12$ (ii) $n(B \times A) = 12$ (iii) $n\{(A \times B) \cap (B \times A)\} = 4$
8. ϕ, $\{(1, 2)\}$, $\{(1, 3)\}$, $\{(2, 2)\}$, $\{(2, 3)\}$, $\{(1, 2), (1, 3)\}$, $\{(1, 2), (2, 2)\}$, $\{(1, 2), (2, 3)\}$, $\{(1, 3), (2, 2)\}$, $\{(1, 3), (2, 3)\}$, $\{(2, 2), (2, 3)\}$, $\{(1, 2), (1, 3), (2, 2)\}$, $\{(1, 2), (1, 3), (2, 3)\}$, $\{(1, 3), (2, 2), (2, 3)\}$, $\{(1, 2), (2, 2), (2, 3)\}$, $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$

HINTS TO SOME SELECTED QUESTIONS

6. (i) We know that $n(A \times B) = n(A) \times n(B)$ (ii) $n(B \times A) = n(B) \times n(A)$.
   (iii) Since A and B have 2 elements in common, so $(A \times B)$ and $(B \times A)$ will have $2^2 = 4$ elements in common.
7. We know that $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A) = (A \cap B) \times (A \cap B)$.
   Now, $n(A \cap B) = 1 \Leftrightarrow n\{(A \cap B) \times (A \cap B)\} = 1^2 = 1$
   $\Leftrightarrow n\{(A \times B) \cap (B \times A)\} = 1 [\because (A \cap B) \times (A \cap B) = (A \times B) \cap (B \times A)]$.
8. $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$. So, it will have $2^4 = 16$ subsets.

RELATIONS

RELATION *Let A and B be two nonempty sets. Then, a relation R from A to B is a subset of* $(A \times B)$.

Thus, R is a relation from A to B $\Leftrightarrow R \subseteq (A \times B)$.

If $(a, b) \in R$ then we say that '*a is related to b*' and we write, $a \, R \, b$.

If $(a, b) \notin R$ then '*a is not related to b*' and we write, $a \not{R} \, b$.

DOMAIN, RANGE AND CO-DOMAIN OF A RELATION

Let *R* be a relation from *A* to *B*. Then, $R \subseteq (A \times B)$.

(i) *The set of all first coordinates of elements of R is called the domain of R, written as dom (R).*

(ii) *The set of all second coordinates of elements of R is called the range of R, denoted by range (R).*

(iii) *The set B is called the co-domain of R.*

Dom $(R) = \{a : (a, b) \in R\}$ and *Range* $(R) = \{b : (a, b) \in R\}$.

Total Number of Relations from *A* to *B*

Let $n(A) = p$ and $n(B) = q$. Then, $n(A \times B) = pq$.

We know that every subset of $A \times B$ is a relation from *A* to *B*.

Total number of subsets of $A \times B$ is 2^{pq}.

$\therefore$ *total number of relations from A to* $B = 2^{pq}$.

REPRESENTATION OF A RELATION

Let *A* and *B* be two given sets. Then, a relation $R \subseteq A \times B$ can be represented in any of the forms, given below.

(i) ROSTER FORM

In this form, *R* is given as a set of ordered pairs.

EXAMPLE 1 *Let* $A = \{-2, -1, 0, 1, 2\}$ *and* $B = \{0, 1, 4, 9\}$.

Let $R = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$.

(i) Show that R is a relation from A to B.

(ii) Find dom (R), range (R) and co-domain of R.

SOLUTION (i) Since $R \subset A \times B$, so *R* is a relation from *A* to *B*.

Note that $-2R4, -1R1, 0R0, 1R1$ and $2R4$.

(ii) Dom (R) = set of first coordinates of elements of *R*

$= \{-2, -1, 0, 1, 2\}$.

Range (R) = set of second coordinates of elements of *R*

$= \{0, 1, 4\}$.

Co-domain of $R = \{0, 1, 4, 9\} = B$.

(ii) SET-BUILDER FORM

Under this method, for every $(a, b) \in R$, *a general relation is being given between a and b.*

Using this relation, all the elements of R can be obtained.

EXAMPLE 2 *Let* $A = \{1, 2, 3, 5\}$ *and* $B = \{4, 6, 9\}$.

Define a relation from A to B, given by

$$R = \{(a, b) : a \in A, b \in B \text{ and } (a - b) \text{ is odd}\}.$$

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

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SOLUTION (i) Clearly, we have

$$R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}.$$

(ii) Dom (R) = set of first coordinates of elements of R

$= \{1, 2, 3, 5\}.$

Range (R) = set of second coordinates of elements of R

$= \{4, 6, 9\}.$

EXAMPLE 3 *Let $R = \{(x, x^3) : x$ is a prime number less than 10$\}$.*

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

SOLUTION Prime numbers less than 10 are 2, 3, 5, 7.

(i) $R = \{(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)\}$

$= \{(2, 8), (3, 27), (5, 125), (7, 343)\}.$

(ii) Dom $(R) = \{2, 3, 5, 7\}.$

Range $(R) = \{8, 27, 125, 343\}.$

EXAMPLE 4 *Let $R = \{(x, y) : x$ and y are integers and $xy = 4\}$.*

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

SOLUTION Clearly, we have

(i) $R = \{(-4, -1), (-2, -2), (-1, -4), (1, 4), (2, 2), (4, 1)\}.$

(ii) Dom $(R) = \{-4, -2, -1, 1, 2, 4\}.$

Range $(R) = \{-4, -2, -1, 1, 2, 4\}.$

(iii) ARROW DIAGRAM

Let R be a relation from A to B. First, we draw two bounded figures to represent A and B respectively. We mark the elements of A and B in these figures. For each $(a, b) \in R$, we draw an arrow from a to b. This gives us the required arrow diagram.

EXAMPLE 5 *Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R from A to A by $R = \{(x, y) : y = 2x - 3\}$.*

(i) Depict R using an arrow diagram.

(ii) Find dom (R) and range (R).

SOLUTION $x = 1 \Rightarrow y = (2 - 3) = -1 \notin A$

$x = 2 \Rightarrow y = (4 - 3) = 1,$

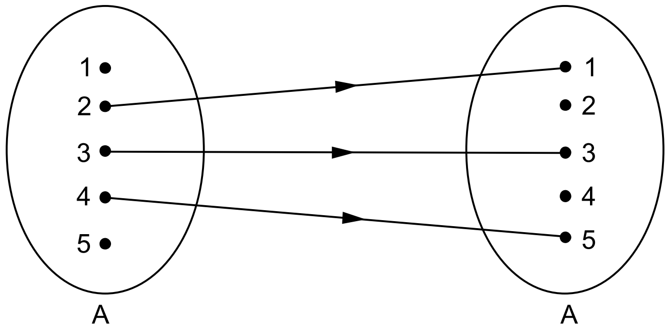
$x = 3 \Rightarrow y = (6 - 3) = 3,$

$x = 4 \Rightarrow y = (8 - 3) = 5,$

$x = 5 \Rightarrow y = (10 - 3) = 7 \notin A.$

$\therefore \quad R = \{(2, 1), (3, 3), (4, 5)\}.$

(i) We may depict it by arrow diagram, as shown below:



(ii) We have, dom $(R) = \{2, 3, 4\}$ and range $(R) = \{1, 3, 5\}$.

EXAMPLE 6 *Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 4, 5\}$.*

Let R be a relation 'is less than' from A to B.

(i) List the elements of R.

(ii) Find the domain, co-domain and range of R.

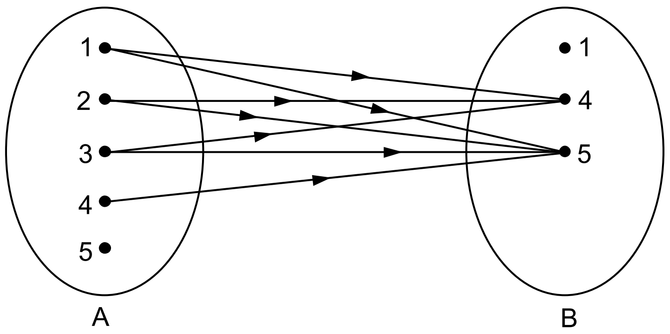
(iii) Depict the above relation by an arrow diagram.

SOLUTION Here, $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 4, 5\}$.

(i) $R = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$.

(ii) Dom $(R) = \{1, 2, 3, 4\}$, range $(R) = \{4, 5\}$
and co-domain $(R) = \{1, 4, 5\}$.

(iii) We may represent the above relation by an arrow diagram, shown below.



REMARK Let *A* and *B* be two nonempty sets. Then, every subset of $A \times B$ is a relation from *A* to *B*.

Since $\phi \subset A \times B$, so ϕ is also a relation from *A* to *B*, called an *empty* or *void* relation.

EXAMPLE 7 *Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of all possible relations that can be defined from A to B.*

SOLUTION Here $n(A) = 3$ and $n(B) = 2$. So, $n(A \times B) = (3 \times 2) = 6$.

Since a set containing *n* elements has 2^n subsets.

$\therefore$ $(A \times B)$ will have $2^6 = 64$ subsets.

But, every subset of $A \times B$ is a relation from *A* to *B*.

Hence, there are in all 64 relations from *A* to *B*.

EXERCISE 2C

1. Let A and B be two nonempty sets.

(i) What do you mean by a relation from A to B?

(ii) What do you mean by the domain and range of a relation?

2. Find the domain and range of each of the relations given below:

(i) $R = \{(-1, 1), (1, 1), (-2, 4), (2, 4), (3, 9)\}$

(ii) $R = \left\{\left(x, \frac{1}{x}\right) : x \text{ is an integer}, 0 < x < 5\right\}$

(iii) $R = \{(x, y) : x + 2y = 8 \text{ and } x, y \in N\}$

(iv) $R = \{(x, y) : y = |x - 1|, x \in Z \text{ and } |x| \leq 3\}$

3. Let $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$.

Let $R = \{(x, y) : x \in A, y \in B$ and $x > y\}$.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

(iii) Depict R by an arrow diagram.

4. Let $A = \{2, 4, 5\}$ and $B = \{1, 2, 3, 4, 6, 8\}$.

Let $R = \{(x, y) \; x \in A, y \in B$ and x divides $y\}$.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

5. Let $A = \{2, 3, 4, 5\}$ and $B = \{3, 6, 7, 10\}$.

Let $R = \{(x, y) : x \in A, y \in B$ and x is relatively prime to $y\}$.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

6. Let $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$.

Let $R = \{(x, y) : x, \in A, y \in B$ and $(x - y)$ is odd$\}$.

Write R in roster form.

7. Let $R = \{(x, y) : x + 3y = 12, x \in N$ and $y \in N\}$.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

8. Let $A = \{1, 2, 3, 4, 5, 6\}$.

Define a relation R from A to A by $R = \{(x, y) : y = x + 1\}$.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

(iii) What is its co-domain?

(iv) Depict R by using arrow diagram.

9. Let $R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

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10. Let $A = \{1, 2, 3, 4, 6\}$ and let $R = \{(a, b) : a, b \in A$ and *a* divides *b*}.

(i) Write *R* in roster form.

(ii) Find dom (*R*) and range (*R*).

11. Define a relation *R* from *Z* to *Z*, given by

$R = \{(a, b) : a, b \in Z$ and $(a - b)$ is an integer}.

Find dom (*R*) and range (*R*).

Hint The difference of two integers is always an integer.

12. Let $R = \{(x, y) : x, y \in Z \text{ and } x^2 + y^2 \le 4\}$.

(i) Write *R* in roster form.

(ii) Find dom (*R*) and range (*R*).

13. Let $A = \{2, 3\}$ and $B = \{3, 5\}$.

(i) Find $(A \times B)$ and $n(A \times B)$.

(ii) How many relations can be defined from *A* to *B*?

14. Let $A = \{3, 5\}$ and $B = \{7, 9\}$. Let $R = \{(a, b) : a \in A, b \in B$ and $(a - b)$ is odd}. Show that *R* is an empty relation from *A* to *B*.

Hint The difference of two odd numbers cannot be odd.

ANSWERS (EXERCISE 2C)

2. (i) dom $(R) = \{-2, -1, 1, 2, 3\}$ and range $(R) = \{1, 4, 9\}$

(ii) dom $(R) = \{1, 2, 3, 4\}$ and range $(R) = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

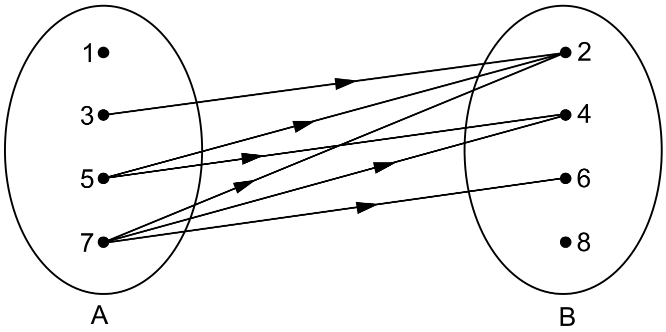
(iii) dom $(R) = \{2, 4, 6\}$ and range $(R) = \{3, 2, 1\}$

(iv) dom $(R) = \{-3, -2, -1, 0, 1, 2, 3\}$ and range $(R) = \{0, 1, 2, 3, 4\}$

3. (i) $R = \{(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)\}$

(ii) dom $(R) = \{3, 5, 7\}$ and range $(R) = \{2, 4, 6\}$

(iii)



4. (i) $R = \{(2, 2), (2, 4), (2, 6), (2, 8), (4, 4), (4, 8)\}$

(ii) dom $(R) = \{2, 4\}$ and range $(R) = \{2, 4, 6, 8\}$

5. (i) $R = \{(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)\}$

(ii) dom $(R) = \{2, 3, 4, 5\}$ and range $(R) = \{3, 6, 7, 10\}$

6. $R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

7. (i) $R = \{(3, 3), (6, 2), (9, 1)\}$

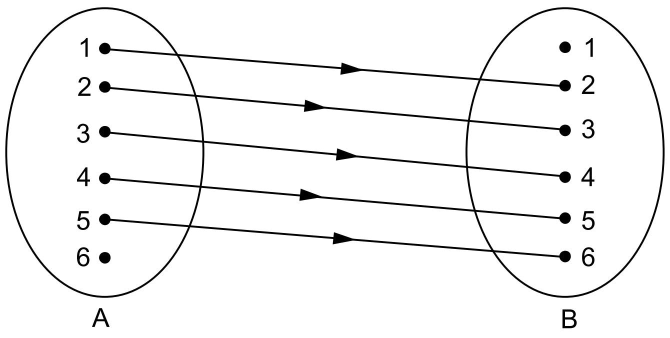
(ii) dom $(R) = \{3, 6, 9\}$ and range $(R) = \{3, 2, 1\}$

8. (i) $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

(ii) dom $(R) = \{1, 2, 3, 4, 5\}$, range $(R) = \{2, 3, 4, 5, 6\}$

(iii) co-domain of $R = \{1, 2, 3, 4, 5, 6\}$

(iv)



9. (i) $R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), 5, 10)\}$

(ii) dom $(R) = \{0, 1, 2, 3, 4, 5\}$ and range $(R) = \{5, 6, 7, 8, 9, 10\}$

10. (i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6),$
$(4, 4), (6, 6)\}$

(ii) dom $(R) = \{1, 2, 3, 4, 6\}$ and range $(R) = \{1, 2, 3, 4, 6\}$

11. dom $(R) = Z$ and range $(R) = Z$

12. (i) $R = \{(-2, 0), (0, -2), (2, 0), (0, 2), (-1, 0), (0, -1), (1, 0), (0, 1), (1, 1),$
$(1, -1), (-1, 1), (-1, -1)\}$

(ii) dom $(R) = \{-2, -1, 0, 1, 2\}$ and range $(R) = \{-2, -1, 0, 1, 2\}$

13. (i) $A \times B = \{(2, 3), (2, 5), (3, 3), (3, 5)\}$ and $n(A \times B) = 4$

(ii) number of relations from *A* to $B = 2^4 = 16$

BINARY RELATION ON A SET

Let A be a nonempty set. Then, every subset of $(A \times A)$ is called a binary relation or simply a relation on A.

REMARKS (i) Since $\phi \subset A \times A$ so ϕ is relation on *A*, called the *empty* or *void relation* on *A*.

(ii) Since $A \times A \subseteq A \times A$ so $(A \times A)$ is a relation on *A*, called the *universal relation* on *A*.

(iii) Let $I_A = \{(a, a) : a \in A\}$. Then, clearly, $I_A \subseteq (A \times A)$ and therefore, it is a relation on *A*, called the *identity relation* on *A*.

SOLVED EXAMPLES

EXAMPLE 1 *Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (3, 2)\}$. Show that R is a binary relation on A. Find its domain and range.*

SOLUTION $R = \{(1, 2), (2, 2), (3, 1), (3, 2)\}$.

Clearly, $R \subset A \times A$ and so *R* is a relation on *A*.

Dom (R) = set of first coordinates of elements of $R = \{1, 2, 3\}$.

Range (R) = set of second coordinates of elements of $R = \{1, 2\}$.

Hence, dom $(R) = \{1, 2, 3\}$ and range $(R) = \{1, 2\}$.

EXAMPLE 2 *Let N be the set of all natural numbers. Let* $R = \{(a, b) : a, b \in N$ *and* $2a + b = 10\}$. *Show that R is a binary relation on N. Find its domain, range and co-domain.*

SOLUTION Here $R = \{(a, b) : a, b \in N$ and $2a + b = 10\}$.

Now, $2a + b = 10 \Rightarrow b = (10 - 2a)$.

$\therefore$ $(a = 1 \Rightarrow b = 8), (a = 2 \Rightarrow b = 6), (a = 3 \Rightarrow b = 4), (a = 4 \Rightarrow b = 2)$.

$\therefore$ $R = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$.

Since $R \subset N \times N$, so *R* is a binary relation on *N*.

Dom (*R*) = set of 1st coordinates of elements of *R*

$= \{1, 2, 3, 4\}$.

Range (*R*) = set of 2nd coordinates of elements of *R*

$= \{8, 6, 4, 2\}$.

Co-domain of $R = N$.

INVERSE RELATION

Let R be a binary relation on a set A. Then, the inverse of R, denoted by R^{-1} *is a binary relation on A, defined by* $R^{-1} = \{(b, a) : (a, b) \in R\}$.

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$.

Also, ***dom* (R) = *range* (R^{-1}) *and range* (R) = *dom* (R^{-1}).**

EXAMPLE 3 *Let A be the set of first ten natural numbers. Let R be a binary relation on A, defined by*

$R = \{(a, b) : a, b \in A \text{ and } a + 2b = 10\}$.

Express R and R^{-1} *as sets of ordered pairs.*

Show that (i) dom (R) = range(R^{-1}) *(ii) range (R) = dom* (R^{-1}).

SOLUTION $a + 2b = 10 \Rightarrow b = \dfrac{(10 - a)}{2}$.

Now, $(a = 2 \Rightarrow b = 4), (a = 4 \Rightarrow b = 3), (a = 6 \Rightarrow b = 2),$
$(a = 8 \Rightarrow b = 1)$.

$\therefore$ $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$

$\Rightarrow$ $R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$.

$\therefore$ (i) dom $(R) = \{2, 4, 6, 8\}$ = range (R^{-1}).

(ii) range $(R) = \{4, 3, 2, 1\}$ = dom (R^{-1}).

VARIOUS TYPES OF RELATIONS

Let *A* be a nonempty set. Then, a relation *R* on *A* is said to be:

(i) *reflexive*, if $(a, a) \in R$ for all $a \in A$, i.e., *a R a* for all $a \in A$;

(ii) *symmetric*, if $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$,

i.e., $a\,R\,b \Rightarrow b\,R\,a$ for all $a, b \in A$;

(iii) *transitive*, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.
i.e., $a\,R\,b, b\,R\,c \Rightarrow a\,R\,c$ for all $a, b, c \in A$.

EQUIVALENCE RELATION

A relation which is reflexive, symmetric and transitive is called an equivalence relation.

EXAMPLE 4 *Let R be a relation on the set Q of all rationals defined by* $R = \{(a, b) : a, b \in Q$ *and* $a - b \in Z\}$. *Show that R is an equivalence relation.*

SOLUTION Given: $R = \{(a, b) : a, b \in Q$ and $a - b \in Z\}$.

(i) Let $a \in Q$. Then, $a - a = 0 \in Z$.

$\therefore$ $(a, a) \in R$ for all $a \in Q$.

So, *R* is reflexive.

(ii) $(a, b) \in R \Rightarrow (a - b) \in Z$, i.e., $(a - b)$ is an integer

$\Rightarrow -(a - b)$ is an integer

$\Rightarrow (b - a)$ is an integer

$\Rightarrow (b, a) \in R$.

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$.

$\therefore$ *R* is symmetric.

(iii) $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $(a - b)$ is an integer and $(b - c)$ is an integer

$\Rightarrow$ $\{(a - b) + (b - c)\}$ is an integer

$\Rightarrow$ $(a - c)$ is an integer

$\Rightarrow$ $(a, c) \in R$.

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

$\therefore$ *R* is transitive.

Thus, *R* is reflexive, symmetric and transitive.

So, *R* is an equivalence relation.

EXAMPLE 5 *Let m be a given fixed positive integer.*

Let $R = \{(a, b) : a, b \in Z$ *and* $(a - b)$ *is divisible by m*$\}$.

Show that R is an equivalence relation on Z.

SOLUTION $R = \{(a, b) : a, b \in Z$ and $(a - b)$ is divisible by *m*$\}$.

(i) Let $a \in Z$. Then,

$a - a = 0$, which is divisible by *m*.

$\therefore$ $(a, a) \in R$ for all $a \in Z$.

So, *R* is reflexive.

(ii) Let $(a, b) \in R$. Then,

$(a, b) \in R \Rightarrow (a - b)$ is divisible by *m*

$\Rightarrow -(a - b)$ is divisible by *m*

$\Rightarrow (b - a)$ is divisible by *m*

$\Rightarrow (b, a) \in R$.

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$.

So, *R* is symmetric.

(iii) Let $(a, b) \in R$ and $(b, c) \in R$. Then,

$(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow$ $(a - b)$ is divisible by *m* and $(b - c)$ is divisible by *m*

$\Rightarrow$ $\{(a - b) + (b - c)\}$ is divisible by *m*

$\Rightarrow$ $(a - c)$ is divisible by *m*

$\Rightarrow$ $(a, c) \in R$.

$\therefore$ $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

So, *R* is transitive.

Thus, *R* is reflexive, symmetric and transitive.

Hence, *R* is an equivalence relation on *Z*.

EXAMPLE 6 *Show that the relation 'is parallel to' on the set S of all straight lines in a plane is an equivalence relation.*

SOLUTION Let *S* be the set of all straight lines in a plane.

Then, the relation, 'is parallel to' on *S* is

(i) *reflexive*, since every line is parallel to itself,

i.e., $L \parallel L$ for all *L* in *S*;

(ii) *symmetric*, since $L \parallel M \Rightarrow M \parallel L$ for all $L, M \in S$;

(iii) *transitive*, since for all *L*, *M*, *N* in *S*, we have

$L \parallel M$ and $M \parallel N \Rightarrow L \parallel N$.

Thus, the given relation is reflexive, symmetric and transitive.

Hence, it is an equivalence relation on *S*.

EXAMPLE 7 *Show that the relation 'is congruent to' on the set of all triangles in a plane is an equivalence relation.*

SOLUTION Let *S* be the set of all triangles in a plane.

Then, the congruence relation on *S* is

(i) *reflexive*, since $\triangle \cong \triangle$ for every $\triangle \in S$;

(ii) *symmetric*, since $\triangle_1 \cong \triangle_2 \Rightarrow \triangle_2 \cong \triangle_1$ for all $\triangle_1, \triangle_2 \in S$

(iii) *transitive*, since $\triangle_1 \cong \triangle_2$ and $\triangle_2 \cong \triangle_3 \Rightarrow \triangle_1 \cong \triangle_3$

for all $\triangle_1, \triangle_2, \triangle_3 \in S$.

Hence, the given relation is an equivalence relation.

EXAMPLE 8 *Let* $R = \{(a, b) : a, b \in N$ *and* $a = b^2\}$. *Show that R satisfies none of reflexivity, symmetry and transitivity.*

SOLUTION (i) *R* is not reflexive, since $2 \neq 2^2$ and therefore $(2, 2) \notin R$.

(ii) Since $4 = 2^2$, so $(4, 2) \in R$.

But, $2 \neq 4^2$. So, $(2, 4) \notin R$.

Thus, $(4, 2) \in R$ but $(2, 4) \notin R$.

$\therefore$ *R* is not symmetric.

(iii) Since $16 = 4^2$, so $(16, 4) \in R$.

Also, $4 = 2^2$, so $(4, 2) \in R$.

But, $16 \neq 2^2 \Rightarrow (16, 2) \notin R$.

Thus, $(16, 4) \in R$ and $(4, 2) \in R$. But $(16, 2) \notin R$.

$\therefore$ R is not transitive.

Hence, R satisfies none of reflexivity, symmetry and transitivity.

EXAMPLE 9 *Let S be the set of all real numbers and let R be a binary relation on S defined by $(a, b) \in R \Leftrightarrow 1 + ab > 0$ for all $a, b \in S$. Show that R is reflexive as well as symmetric. Give an example to show that R is not transitive.*

SOLUTION (i) Let a be an arbitrary real number.

Then, $(1 + a^2) > 0 \Rightarrow (a, a) \in R$ for every $a \in S$.

$\therefore$ R is reflexive.

(ii) Let $(a, b) \in R$. Then,

$$\begin{aligned} (a, b) \in R &\Rightarrow 1 + ab > 0 \\ &\Rightarrow 1 + ba > 0 \\ &\Rightarrow (b, a) \in R. \end{aligned}$$

$\therefore$ $(a, b) \in R \Rightarrow (b, a) \in R$.

So, R is symmetric.

(iii) In order to show that R is not transitive, consider the real numbers $\left(\frac{-2}{3}\right)$, 1 and 2.

Now, $\left(\frac{-2}{3}, 1\right) \in R$, since $\left\{1 + \left(\frac{-2}{3}\right) \times 1\right\} = \left(1 - \frac{2}{3}\right) = \frac{1}{3} > 0$.

And, $(1, 2) \in R$, since $\{1 + (1 \times 2)\} = 3 > 0$.

But, $\left(\frac{-2}{3}, 2\right) \notin R$, since $\left\{1 + \left(\frac{-2}{3}\right) \times 2\right\} = \left(1 - \frac{4}{3}\right) = -\frac{1}{3} < 0$.

Thus, $\left(\frac{-2}{3}, 1\right) \in R$, $(1, 2) \in R$ and $\left(\frac{-2}{3}, 2\right) \notin R$.

This shows that R is not transitive.

EXAMPLE 10 *Let R be a relation on $N \times N$, defined by*

$$(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c \text{ for all } (a, b), (c, d) \in N \times N.$$

Show that R is an equivalence relation.

SOLUTION Here R is a relation on $N \times N$, defined by

$$(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c \text{ for all } (a, b), (c, d) \in N \times N.$$

We shall show that R satisfies the following properties.

(i) **Reflexivity:**

We know that $a + b = b + a$ for all $a, b \in N$.

$\therefore$ $(a, b)\ R\ (a, b)$ for all $(a, b) \in (N \times N)$.

So, R is reflexive.

(ii) **Symmetry:**

Let $(a, b)\, R\, (c, d)$. Then,

$$(a, b)\, R\, (c, d) \Rightarrow a + d = b + c$$
$$\Rightarrow c + b = d + a$$
$$\Rightarrow (c, d)\, R\, (a, b).$$

$\therefore$ $(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for all $(a, b), (c, d) \in N \times N$.

This shows that *R* is symmetric.

(iii) **Transitivity:**

Let $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$. Then,

$(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

$\Rightarrow$ $a + d = b + c$ and $c + f = d + e$

$\Rightarrow$ $a + d + c + f = b + c + d + e$

$\Rightarrow$ $a + f = b + e$

$\Rightarrow$ $(a, b)\, R\, (e, f)$.

Thus, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f) \Rightarrow (a, b)\, R\, (e, f)$.

This shows that *R* is transitive.

$\therefore$ *R* is reflexive, symmetric and transitive.

Hence, *R* is an equivalence relation on $N \times N$.

EXERCISE 2D

1. What do you mean by a binary relation on a set *A*?
Define the domain and range of a relation on *A*.

2. Let $A = \{2, 3, 5\}$ and $R = \{(2, 3), (2, 5), (3, 3), (3, 5)\}$.
Show that *R* is a binary relation on *A*. Find its domain and range.

3. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and let $R = \{(a, b) : a, b \in A \text{ and } 2a + 3b = 12\}$.
Express *R* as a set of ordered pairs. Show that *R* is a binary relation on *A*.
Find its domain and range.

4. If *R* is a binary relation on a set *A*, define R^{-1} on *A*.
Let $R = \{(a, b) : a, b \in W \text{ and } 3a + 2b = 15\}$, where *W* is the set of whole numbers.
Express *R* and R^{-1} as sets of ordered pairs.
Show that (i) dom (*R*) = range (R^{-1}) (ii) range (*R*) = dom (R^{-1}).

5. What is an equivalence relation?
Show that the relation of 'similarity' on the set *S* of all triangles in a plane is an equivalence relation.

6. Let $R = \{(a, b) : a, b \in Z \text{ and } (a - b) \text{ is even}\}$.
Then, show that *R* is an equivalence relation on *Z*.

7. Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : a, b \in A \text{ and } |a^2 - b^2| \le 5$.
Write *R* as set of ordered pairs.

Mention whether *R* is (i) reflexive (ii) symmetric (iii) transitive.
Give reason in each case.

8. Let $R = \{(a, b) : a, b \in Z$ and $b = 2a - 4\}$. If $(a, -2) \in R$ and $(4, b^2) \in R$. Then, write the values of *a* and *b*.

9. Let *R* be a relation on *Z*, defined by $(x, y) \in R \Leftrightarrow x^2 + y^2 = 9$. Then, write *R* as set of ordered pairs. What is its domain?

10. Let *A* be the set of first five natural numbers and let *R* be a relation on *A*, defined by $(x, y) \in R \Leftrightarrow x \leq y$.
Express *R* and R^{-1} as sets of ordered pairs.
Find: dom (R^{-1}) and range (*R*).

11. Let $R = \{(x, y) : x, y \in Z$ and $x^2 + y^2 = 25\}$.
Express *R* and R^{-1} as sets of ordered pairs. Show that $R = R^{-1}$.

12. Find R^{-1}, when
(i) $R = \{(1, 2), (1, 3), (2, 3), (3, 2), (4, 5)\}$,
(ii) $R = \{(x, y) : x, y \in N, x + 2y = 8\}$.

13. Let $A = \{a, b\}$. List all relations on *A* and find their number.

Hint $A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$ and every subset of $A \times A$ is a relation on *A*.
So, their number $= 2^4 = 16$.

14. Let $R = \{(a, b) : a, b \in N$ and $a < b\}$.
Show that *R* is a binary relation on *N*, which is neither reflexive nor symmetric. Show that *R* is transitive.

Hint Since $R \subset N \times N$, so it is a binary relation on *N*.

ANSWERS (EXERCISE 2D)

2. dom $(R) = \{2, 3\}$ and range $(R) = \{3, 5\}$

3. $R = \{(0, 4), (3, 2), (6, 0)\}$, dom $(R) = \{0, 3, 6\}$ and range $(R) = \{0, 2, 4\}$

4. $R = \{(1, 6), (3, 3), (5, 0)\}$, $R^{-1} = \{(6, 1), (3, 3), (0, 5)\}$

7. $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (3, 2), (2, 3)\}$.
R is reflexive and symmetric but not transitive.

8. $a = 1, b = 4$

9. $R = \{(-3, 0), (0, -3), (3, 0), (0, 3)$, dom $(R) = \{-3, 0, 3\}$

10. $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)\}$
$R^{-1} = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (2, 2), (3, 2), (4, 2), (5, 2), (3, 3), (4, 3), (5, 3), (4, 4), (5, 4), (5, 5)\}$
dom $(R^{-1}) = \{1, 2, 3, 4, 5\} = \text{range } (R)$

11. $R = \{(-5, 0), (0, -5), (5, 0), (0, 5), (3, 4), (4, 3), (-3, 4), (4, -3), (-3, -4), (-4, -3)\}$
$R^{-1} = \{(0, -5), (-5, 0), (0, 5), (5, 0), (4, 3), (3, 4), (4, -3), (-3, 4), (-4, -3), (-3, -4)\}$

12. (i) $R^{-1} = \{(2,1),(3,1),(3,2),(2,3),(5,4)\}$ (ii) $R^{-1} = \{(3,2),(2,4),(1,6)\}$

13. 16

EXERCISE 2E

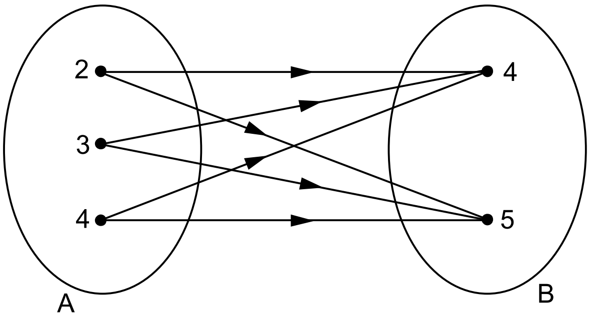
Very-Short-Answer Questions

1. Let *A* and *B* be two sets such that $n(A) = 5$, $n(B) = 3$ and $n(A \cap B) = 2$.
   (i) $n(A \cup B)$ (ii) $n(A \times B)$ (iii) $n\{(A \times B) \cap (B \times A)\}$

   Hint (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
   (ii) $n(A \times B) = n(A) \cdot n(B)$
   (iii) If $n(A \cap B) = m$ then $n\{(A \times B) \cap (B \times A)\} = 2^m$.
2. Find *a* and *b* when $(a - 2b, 13) = (7, 2a - 3b)$.
3. If $A = \{1, 2\}$, find $A \times A \times A$.
4. If $A = \{2, 3, 4\}$ and $B = \{4, 5\}$, draw an arrow diagram to represent $(A \times B)$.
5. If $A = \{3, 4\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$, find $A \times (B \times C)$.
6. If $A \subseteq B$, prove that $A \times C = B \times C$.
7. Prove that $A \times B = B \times A \Rightarrow A = B$.
8. If $A = \{5\}$ and $B = \{5, 6\}$, write down all possible subsets of $A \times B$.
9. Let $R = \{(x, x^2) : x$ is a prime number less than 10}.
   (i) Write *R* in roster form.
   (ii) Find dom (*R*) and range (*R*).
10. Let $A = \{1, 2, 3\}$ and $B = \{4\}$.
    How many relations can be defined from *A* to *B*?
11. Let $A = \{3, 4, 5, 6\}$ and $R = \{(a, b) : a, b \in A \text{ and } a > b\}$.
    Write *R* in roster form.
    Find: dom (*R*) and range (*R*).
    Write R^{-1} in roster form.
12. Let $R = \{(a, b) : a, b \in N, a > b\}$.
    Show that *R* is a binary relation which is neither reflexive, nor symmetric.
    Show that *R* is transitive.

ANSWERS (EXERCISE 2E)

1. (i) 6 (ii) 15 (iii) 4
2. $a = 5, b = -1$
3. $\{(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1),(2,2,2)\}$
4.



A × B

5. $\{(3, 4, 5), (3, 4, 6), (3, 5, 5), (3, 5, 6), (4, 4, 5), (4, 4, 6), (4, 5, 5), (4, 5, 6)\}$

8. $\phi, \{(5, 5)\}, \{(5, 6)\}, \{(5, 5), (5, 6)\}$

9. (i) $R = \{(2, 4), (3, 9), (5, 25), (7, 49)\}$

(ii) dom $(R) = \{2, 3, 5, 7\}$ and range $(R) = \{4, 9, 25, 49\}$

10. 8

11. $R = \{(4, 3), (5, 3), (5, 4), (6, 3), (6, 4), (6, 5)$

(i) dom $(R) = \{4, 5, 6\}$ and range $(R) = \{3, 4, 5\}$

$R^{-1} = \{(3, 4), (3, 5), (4, 5), (3, 6), (4, 6), (5, 6)\}$

SUMMARY OF KEY FACTS

1. Two numbers *a* and *b* listed in a specific order and enclosed in parentheses form an ordered pair (a, b).
Here *a* is the first component and *b* is the second component.
In general, $(a, b) \neq (b, a)$.

2. $(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$.

3. Let *A* and *B* be two nonempty sets. Then, the Cartesian product of *A* and *B* is defined as

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$$

If $A = \phi$ or $B = \phi$ then $A \times B = \phi$.

4. If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$.

5. Three numbers *a, b, c* listed in a specific order and enclosed in parentheses form an ordered triplet (a, b, c).

$$(a, b, c) \neq (b, a, c) \neq (c, a, b), \text{ etc.}$$

6. For any nonempty sets *A, B, C,* we have

$$(A \times B) \times C = A \times (B \times C), \text{ each denoted by } A \times B \times C.$$

7. For any sets *A, B* and *C*, we have:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

(iv) $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A) = (A \cap B) \times (A \cap B)$

(v) $A \times B = A \times C \Rightarrow B = C$

(vi) $A \subset B \Rightarrow (A \times A) \subset (A \times B) \cap (B \times A)$

(vii) $A \subset B \Rightarrow (A \times C) \subset (B \times C)$

(viii) $A \subset B$ and $C \subset D \Rightarrow (A \times C) \subset (B \times D)$

(ix) $A \times B = B \times A \Leftrightarrow A = B$

8. Let *A* and *B* be two nonempty sets and let $R \subseteq A \times B$.
Then, *R* is called *a relation* from *A* to *B*.
If $(a, b) \in R$, we say that '*a is related to b*' and we write, *a R b*.

If $(a, b) \notin R$, we say that '*a is not related to b*' and we write, $a \not R\, b$.
Dom $(R) = \{a : (a, b) \in R\}$, range $(R) = \{b : (a, b) \in R\}$.

9. We define, $R^{-1} = \{(b, a) : (a, b) \in R\}$,

dom (R) = range (R^{-1}) and range (R) = dom (R^{-1}).

10. Let A be a nonempty set. Then, every subset of $A \times A$ is called a binary relation on A.

11. Let A be a nonempty set and R be a binary relation on A.

Then, R is said to be:

(i) ***reflexive,*** if $(a, a) \in R$ for all $a \in A$;

i.e., $a\,R\,a$ for all $a \in A$.

(ii) ***symmetric,*** if $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$;

i.e., $a\,R\,b \Rightarrow b\,R\,a$ for all $a, b \in A$.

(iii) ***transitive,*** if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$;

i.e., $a\,R\,b$ and $b\,R\,c \Rightarrow a\,R\,c$ for all $a, b, c \in R$.

12. A relation R on A, which is reflexive, symmetric and transitive, is called an equivalence relation on A.